

A Delayed Non-Linear Model-Based Control for an Underactuated System*

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Abstract

This paper discusses the visual feedback control of an underactuated mechanisms under fixed-camera configuration. The control goal is to stabilize the system over a desired static target by using a visual scheme propoused in [4] which basically is a vision system equipped with a fixed camera. We present a control scheme based on the combination of a nonlinear state observer and the visual feedback for an underactuated system, the so-called *Pendubot*, consisting in a double pendulum actuated only at the first joint. The paper ends with the presentation of several simulation results and some guidelines future work are drawn in the conclusion.

1 Introduction

This paper presents the application of modern linear systems theory with visual sensor to control an underactuated mechanical system. In the eighties, the control of robot manipulators was extensively studied. Several control strategies based on passivity, Lyapunov theory, feedback linearization, output regulation, etc. have been developed for the fully actuated case, i.e. systems with the same number of actuators as degree of freedom [6, 10, 22]. The techniques developed for fully actuated robots do not apply directly to the case of underactuated mechanical systems [3, 7, 13, 17, 18, 19, 20, 21, 22, 23, 24]. Underactuated mechanical systems or vehicles are systems with fewer independent control actuators than degrees of freedom to be controlled.

In the last few years, there has been major interest in developing stabilizing algorithms for underactuated mechanical systems. The interest comes

from the need to stabilize systems like ships, underwater vehicles, helicopters, aircraft, airships, hovercrafts, satellites, walking robots, etc, which may be underactuated by design. Actuators are expensive and/or heavy and are therefore avoided in a system design. Other systems may also become underactuated due to actuator failure. A visual solution to actuator failures may be achieved by equipping the underactuated system with visual sensors. The use of visual sensor in feedback control loops with robot manipulators represents an attractive solution to position and motion control [1, 2, 4, 5].

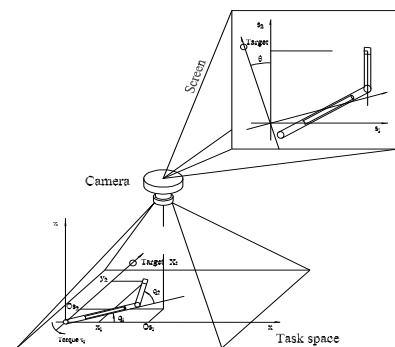


Figure 1: Schematic representation of the Robot-camera system

Most existing generalizations of classical visual servoing techniques exploit a high gain or computed torque feedback to make a dynamic reduction of the system to a controllable kinematic model for which the visual servoing task may be solved directly [5]. The dynamics model of a system is commonly ignored in the design of visual servo systems and closed-loop performance may be severely limited to ensure that the dynamic reduction is valid. Recently in [11] has explored a more nonlinear aspect

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of the system dynamics, and presented an asymptotically stable method for position regulation for fixed-camera visual servoing. The difficulties associated with controlling an underactuated system have received even less attention. In [25] has been working on the visual servoing problem using a Lagrangian representation of the system dynamics and consider underactuated and nonholonomic systems. In [8] proposed a new image-based control strategy for visual feedback which is applicable to a class of underactuated dynamic systems.

The aim of this paper is to introduce the visual feedback in the control of underactuated systems. We develop a dynamic controller using visual sensor for an underactuated dynamic system which operate with accurate target information as shown in Figure 1. The proposed approach is motivated by a theoretical analysis of the dynamic equation of motion of a rigid body and exploits structural linear properties of these dynamics to derive a nonlinear observer and a linear control algorithm.

The paper is organized in the following manner. Section 2 describes the equivalent representation of the robot manipulator model while section 3 is devoted to the nonlinear observer structure. Section 4 gives the Pendubot model, where it is used in Section 5 to design a controller. In section 6 shows some simulation results. Finally, concluding remarks are given in Section 7.

2 Equivalent Representation of the Robot Model

The dynamic equation of an n degree-of-freedom robot manipulator in the continuous time can be written as [22]

$$D(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau \quad (1)$$

where q is the $(n \times 1)$ vector of joint variables (generalized coordinates), $D(q)$ is the $(n \times n)$ symmetric positive-definite inertia matrix, $C(q, \dot{q})$ is the vector of Coriolis and centripetal torques, $G(q)$ are the gravitational terms, and τ is the $(n \times 1)$ vector of input torques.

Choosing as state vector $x = (x_1^T \ x_2^T)^T = (q^T \ \dot{q}^T)^T$, as input $u = \tau$, the description of the system can be given in state space form as:

$$\dot{x}_1 = x_2 \quad (2)$$

$$\dot{x}_2 = -D^{-1}(x_1)[C(x_1, x_2) + G(x_1)] + D^{-1}(x_1)u \quad (3)$$

$$y = Cx \quad (4)$$

or

$$\dot{x} = Ax + B(x)u + d(x) \quad (5)$$

$$y = Cx \quad (6)$$

where

$$A = \begin{bmatrix} \mathcal{O}_{n \times n} & \mathcal{I}_{n \times n} \\ \mathcal{O}_{n \times n} & \mathcal{O}_{n \times n} \end{bmatrix}, \quad B(x) = \begin{bmatrix} \mathcal{O}_{n \times n} \\ D^{-1}(x_1) \end{bmatrix},$$

$$d(x) = \begin{bmatrix} \mathcal{O}_{n \times n} \\ -D^{-1}(x_1)[C(x_1, x_2) + G(x_1)] \end{bmatrix},$$

$$C = [0 \ 1 \ 0 \ 0].$$

where $\mathcal{O}_{n \times n}$ is the $(n \times n)$ null matrix, $\mathcal{I}_{n \times n}$ is the $(n \times n)$ identity matrix and y is the output signal.

2.1 Discrete-Time State-Space Equation

Visual feedback employs discrete-time model. Robot discrete-time dynamics has been studied by many researchers [12, 14, 15, 16]. To obtain a discrete-time state-space equation from a continuous-time state-space equation (5)-(6), we assume that all the measurements of the manipulator state are available at a sampling rate T , and the input torques are maintained constant between the sampling instants, i.e. over each time interval $\mathcal{I}_k = [kT \ (k+1)T]$, where $k \geq 0$ is an integer, for sufficiently small time intervals $\dot{x}$ can be approximated with a first forward difference, as follows:

$$\dot{x} \approx \frac{x(t+T) - x(t)}{T} \quad (7)$$

Thus, the differential equation (5) can be expressed as (approximately)

$$\frac{x(t+T) - x(t)}{T} = Ax + B(x)u + d(x) \quad (8)$$

Solving this equation for $x(t+T)$, we obtain

$$x(t+T) = (I + TA)x(t) + TB(x(t))u(t) + Td(x(t)) \quad (9)$$

Evaluation of equations (9) and (6) at $t = kT$ yields a simple discrete-time model, based on the first order Euler method

$$x[(k+1)T] = \Phi x(kT) + \Gamma(x(kT))u(kT) + \Upsilon(x(kT)) \quad (10)$$

$$y(kT) = Cx(kT) \quad (11)$$

where

$$\Phi = (I + TA) \quad \Gamma = TB \quad \Upsilon = Td$$

3 State Nonlinear Observer Design

We consider the problem of estimating the current state $\mathbf{x}(kT)$ of a nonlinear discrete-time dynamical system, described by a system of first-order-difference equations

$$\begin{aligned} \mathbf{x}[(k+1)T] &= \Phi \mathbf{x}(kT) + \Gamma(\mathbf{x}(kT))u(kT) \\ &\quad + \Upsilon(\mathbf{x}(kT)) \end{aligned} \quad (12)$$

$$\mathbf{y}(kT) = \mathbf{C}\mathbf{x}(kT) \quad (13)$$

from the past observations $\mathbf{y}(sT)$, $s \leq k$, where the discrete-time index $k \in \{0, 1, 2, \dots\}$ and T is the sampling period.

For the discrete-time manipulator model form (12)-(13), the proposed observer is given by

$$\begin{aligned} \hat{\mathbf{x}}[(k+1)T] &= \Phi \hat{\mathbf{x}}(kT) + \Gamma(\hat{\mathbf{x}}(kT))u(kT) \\ &\quad + \Upsilon(\hat{\mathbf{x}}(kT)) + \mathbf{K}_e[\mathbf{y} - \hat{\mathbf{y}}(kT)] \end{aligned} \quad (14)$$

$$\hat{\mathbf{y}}(kT) = \mathbf{C}\hat{\mathbf{x}}(kT) \quad (15)$$

The resulting error equation takes on the following form

$$\mathbf{e}(k+1) = (\Phi - \mathbf{K}_e \mathbf{C})\mathbf{e}, \quad \mathbf{e} = \mathbf{x} - \hat{\mathbf{x}} \quad (16)$$

As the pair $\mathbf{C}, \mathbf{A}$ is observable, the eigenvalues of the error system may be arbitrarily assigned.

4 The PENDUBOT Model

The Pendubot, which is the underactuated system considered here, it is shown schematically in Figure 2. For the purposes of this work, we assume that it has a planar motion without friction.

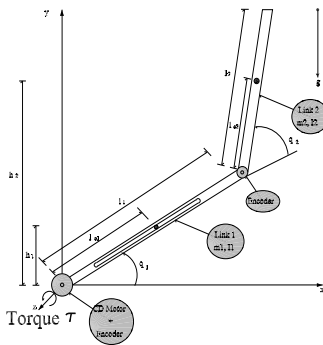


Figure 2: The PENDUBOT system.

For the Pendubot system, the dynamic model (1) is particularized as

Table 1: Parameters of the PENDUBOT.

notation		value	unit
Mass of link 1	m_1	0.5289	kg
Mass of link 2	m_2	0.3346	kg
Length of link 1	l_1	0.26987	m
Length of link 2	l_2	0.38417	m
Distance to the center of mass of link 1	l_{c1}	0.13494	m
Distance to the center of mass of link 2	l_{c2}	0.19208	m
Moment of inertia of link 1 about its centroid	I_1	0.013863	Kgm^2
Moment of inertia of link 2 about its centroid	I_2	0.016749	Kgm^2
Acceleration due to gravity	g	9.81	m/sec^2
Angle that link 1 makes with the horizontal	q_1		rad
Angle that link 2 makes with the link 1	q_2		rad
Torque applied on link 1	τ_1		$Nw - m$

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} \quad (17)$$

where

$$\begin{aligned} D_{11} &= m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos(q_2)) \\ &\quad + I_1 + I_2 \\ D_{12} &= m_2 (l_{c2}^2 + l_1 l_{c2} \cos(q_2)) + I_2 \\ D_{22} &= m_2 l_{c2}^2 + I_2 \\ C_1 &= -2m_2 l_1 l_{c2} \dot{q}_1 \dot{q}_2 \sin(q_2) - m_2 l_1 l_{c2} \dot{q}_2^2 \sin(q_2) \\ C_2 &= m_2 l_1 l_{c2} \dot{q}_1^2 \sin(q_2) \\ G_1 &= m_1 g l_{c1} \cos(q_1) + m_2 g l_1 \cos(q_1) \\ &\quad + m_2 g l_{c2} \cos(q_1 + q_2) \\ G_2 &= m_2 g l_{c2} \cos(q_1 + q_2). \end{aligned}$$

4.1 Equivalent Representation

Choosing as state vector $\mathbf{x} = (\mathbf{x}_1^T \ \mathbf{x}_2^T)^T = (x_1 \ x_2 \ x_3 \ x_4)^T := (q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2)^T = (\mathbf{q}_1^T \ \mathbf{q}_2^T)^T$, as input $u = (\tau_1 \ 0)^T$ and q_2 as the output, the description of the system can be given in state space form (5)-(6), where:

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathbf{B}(\mathbf{x}) &= \frac{1}{\Delta} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ D_{22} & -D_{12} \\ -D_{12} & D_{11} \end{bmatrix}, \\
\mathbf{d}(\mathbf{x}) &= \frac{1}{\Delta} \begin{bmatrix} 0 \\ 0 \\ D_{12}(C_2 + G_2) - D_{12}(C_1 + G_1) \\ D_{12}(C_1 + G_1) - D_{11}(C_2 + G_2) \end{bmatrix}, \\
\mathbf{C} &= [0 \ 1 \ 0 \ 0], \quad \Delta = D_{11}D_{22} - D_{12}^2.
\end{aligned}$$

4.2 Discrete-Time State-Space

For the PENDUBOT model, the matrices Φ , Γ , Υ for discrete-time state-space representation (10)-(11) are

$$\begin{aligned}
\Phi &= \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
\Gamma &= T\mathbf{B} = \frac{T}{\Delta} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ D_{22} & -D_{12} \\ -D_{12} & D_{11} \end{bmatrix}, \\
\Upsilon &= T\mathbf{d} = \frac{T}{\Delta} \begin{bmatrix} 0 \\ 0 \\ D_{12}(C_2 + G_2) - D_{12}(C_1 + G_1) \\ D_{12}(C_1 + G_1) - D_{11}(C_2 + G_2) \end{bmatrix}, \\
\mathbf{C} &= [0 \ 1 \ 0 \ 0].
\end{aligned}$$

Here, $\mathbf{x}(kT)$ is the state vector (4-vector) at k th sampling instant, $u(kT)$ control signal (scalar) at k th sampling and $y(kT) = \mathbf{C}\mathbf{x}(kT)$ is the output at k th sampling. This provides an discrete-time state-space model that can be used in the design of state observers, as discussed in the section that follows.

5 Control Scheme

Our goal is to use an external camera as sensor and use an nonlinear observer to estimate the state of the system and stabilize the Pendubot. In this section it is shown how a visual feedback control may be derived based on estimation techniques for a fix camera with underactuated rigid body dynamics. Visual feedback systems incorporate the visual sensors in the feedback. Figure 3 depicts a block diagram of

the closed-loop control system, this is a block diagram of one degree of freedom (1-DOF). The camera lens is modelled as a simple gain, K_{lens} , which, due to perspective, is a function of target distance. We shall first discuss the full-order state observer and then the state feedback controller.

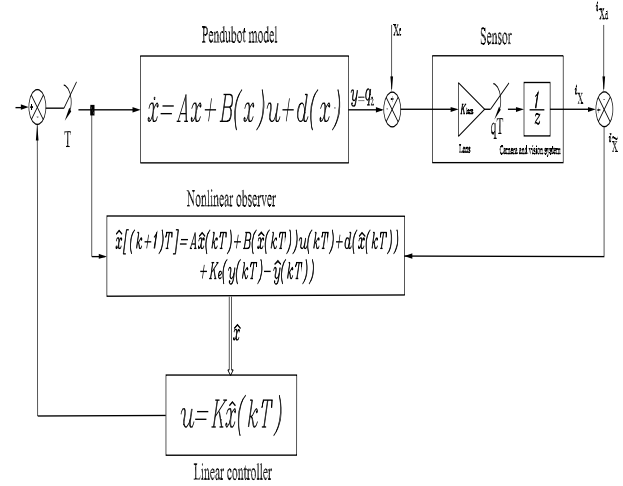


Figure 3: Structure of visual feedback control for the Pendubot. Here, X_t is the world coordinate location of the target, iX_d is the desired location of the target on the image plane, and ${}^i\tilde{X} = {}^iX - {}^iX_d$ is the image plane error.

5.1 State Observer Design

It is important to note that, in the present analysis, state $\mathbf{x}(kT)$ is not available by direct measurement. Since the output $\mathbf{y}(kT) = \mathbf{C}\mathbf{x}(kT)$ can be measured by the fix camera, we can design a state nonlinear observer (14)-(15) for the Pendubot model.

5.2 Controller Design

When the Pendubot is in a neighborhood of its top unstable equilibrium position, a linear controller can stabilize the pendulum quite adequately. We know that the linearized system is observable and controllable, then it is possible to design a linear control [18]. Therefore, the control objective is to stabilize the system around its unstable equilibrium point $\mathbf{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)^T = (\frac{\pi}{2}, 0, 0, 0)^T$, i.e. to bring the second pendulum to its upper position and the first angle q_1 to zero simultaneously. The observed state $\hat{\mathbf{x}}(k)$ is used to form the vector control $\mathbf{u}(k)$, or

$$\mathbf{u}(k) = -\mathbf{K}\hat{\mathbf{x}}(k) \quad (18)$$

where $\mathbf{K}$ is the state feedback gain matrix.

6 Simulation Results

Numerical simulations assuming a discrete-time implementation of the visual controller showed the performance of the closed loop system. In all the simulations, we consider that the initial condition of the system is near to the equilibrium point x^* and the gain K that stabilizes the linear approximation of the Pendubot model was obtained by solving a LQR problem

$$K = \begin{bmatrix} -22.4431 & -21.2982 & -6.2282 & -4.4932 \end{bmatrix}, \quad (19)$$

the observer feedback gain matrix

$$K_e = \begin{bmatrix} -2.9468 \\ 0.3185 \\ -18.6506 \\ 7.7243 \end{bmatrix} \quad (20)$$

and the lens gain

$$K_{lens} = 0.50. \quad (21)$$

We have used *SimulinkTM* and *MATLABTM* to simulate the full dynamic motion of the Pendubot. Figure 4 shows the trajectory of the target in the image plane with ${}^i X_d = 0$, for convenience. The simulation shows that this controller provides a good performance when balancing the links about the unstable vertical position.

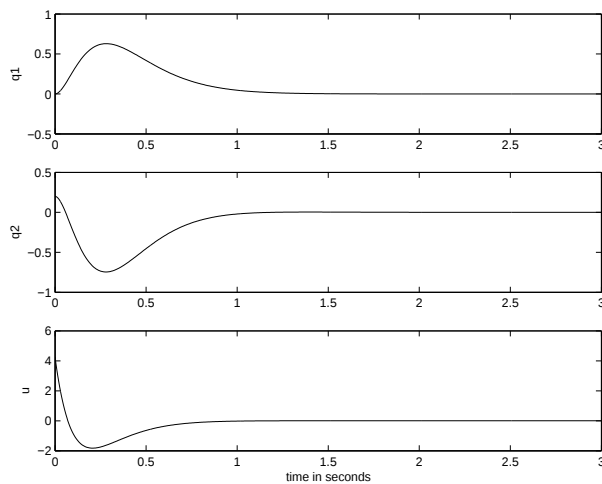


Figure 4: Simulation results. Positioning with respect to the target $X_t = (0, 90^\circ)$.

7 Conclusions and Future Work

This paper presents an alternative approach to the design of discrete-time feedback controllers and nonlinear state observer for an underactuated manipulators using a visual feedback. The case studied is the so-called *Pendubot*, consisting in a double pendulum actuated only at the first joint. The control of the Pendubot is specially difficult since it is an underactuated mechanism (two degrees of freedom and only one input). In this work, we have presented a linear position controller for the Pendubot systems with a fixed camera and fixed target. Specifically, by assuming exact knowledge of the mechanical parameters, and by considering an accepted camera model (as a delay) together with the robot non linear dynamics, we have proposed a visual feedback scheme derived from based on the combination of a nonlinear observer and the visual feedback. Preliminary results indicate that visual feedback is potentially attractive alternative for underactuated systems. An interesting problem is to consider a more realistic camera model. We are currently working to identify a camera model and implement the algorithm on a Texas Instruments TMS320C6711 digital signal processor based system.

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